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PERFORMANCE OF MULTIPLE SATELLITE SYSTEMS WITH COMMON DOWNLINK FREQUENCY HOPPING PATTERNS

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ABSTRACT

This report considers a situation in which more than one satellite transmitting frequency-hopped signal is in view of a given terminal. If it is desired that the satellites use the same frequency hopping pattern for ease of acquisition and handover, then a receiver which is in range of more than one satellite may encounter interference. This interference is analyzed for a generalized system of two satellites modelled after the LES-8/9 system. Upperbounds for the probability of error are derived for receivers using a square-law combining rule. This expression is evaluated for the LES-8/9 system, using two different downlink codes. It is seen that the choice of code makes a significant difference in system performance, and that with a suitably chosen code, two satellites may use the same frequency hopping pattern without significant performance degradation.

CONTENTS

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I. INTRODUCTION

This report considers a situation in which more than one satellite transmitting frequency-hopped signals is in view of a given terminal. If each satellite employs a different frequency hopping pattern, then negligible downlink interference will occur, but a terminal must try multiple hopping patterns during initial acquisition. If it is desired that the satellites use the same frequency hopping pattern for ease of acquisition and handover then a receiver which is in range of more than one satellite may encounter interference. The goal of this study is to characterize this interference and analyze the error probabilities.

The analysis is carried out for a generalized system of two satellites, modeled after the LES-8/9 system [1]. Some attempts were made to investigate a system of more than two satellites, but the equations proved too complex to solve. However, loose bounds for the performance of a multi-satellite system may be obtained by applying the union bound to the results for two satellites.

A. System Model

The signal structure and receiver structure are assumed to be as follows.

One of M equally likely messages, $m_o,\ldots m_{M-1}$, is to be transmitted. Each message waveform consists of a sequence of N channel symbols (or "chips"). Each channel symbol may be one of A orthogonal waveforms having energy E.

In the LES-8/9 system, a binary data stream is divided into groups of three bits each and transmitted as a sequence of one of M=8 messages. Each of the 8 messages consists of N=4 chips. Each chip is a sinusoidal signal of 5 msec duration at frequency f, chosen from one of A=8 frequencies centered around some frequency, f_c. The eight frequencies are $f_c \pm 100$ Hz, $f_c \pm 300$ Hz, $f_c \pm 500$ Hz, and $f_c \pm 700$ Hz. The center frequency is hopped with each chip.

The channel symbols are each received with a random (or unknown) phase shift and an additive white Gaussian noise component with energy density $N_o/2$. The receiver chooses its estimate of the transmitted message by forming M sums:

$$X_m = \sum_{n=1}^{N} e_{mn}^2 \qquad m=0, \ldots, M-1 \tag{1}$$

where e_{mn}^2 is the square of the matched filter envelopes corresponding to the n-th chip of the m-th waveform. The receiver chooses its estimate, $\hat{m}$, to be the message with the greatest filter output, X_m. This is called a square law combining receiver. (It should be noted that this receiver is optimal only for N=1 although it is asymptotically the optimum receiver for larger N at low signal-to-noise ratios.)

Figure 1 shows a receiver for the LES-8/9 signal structure for one chip of one message where the chip frequency is $f_c + \Delta$. This receiver sums the squares of the quadrature components, which gives a filter output equivalent to squaring the signal's envelope.

B. Interfering Signal

Even though the receiver may be in range of both satellites, the unwanted signal may not create interference because it experiences time or frequency shifts with respect to the desired signal. It is assumed that the frequency hopping pattern of the receiver is synchronized in both time and frequency with the hopping pattern of the correct satellite.

If the distances between the receiver and the satellites are such that the difference in propagation times, Δt, is greater than one chip duration, T, and if the center frequencies of successive chips are sufficiently far apart, then the second signal will cause no interference. Since any frequency hopping pattern of practical value must be pseudo-random over a broad band, the second assumption will almost always hold.

In addition, since the satellites will generally be moving at different speeds with respect to the receiver, the signals will experience Doppler shifts. If the difference between the shifted center frequencies of the desired signal and the interfering signal, Δf_c, is sufficiently large, then there will be no interference.

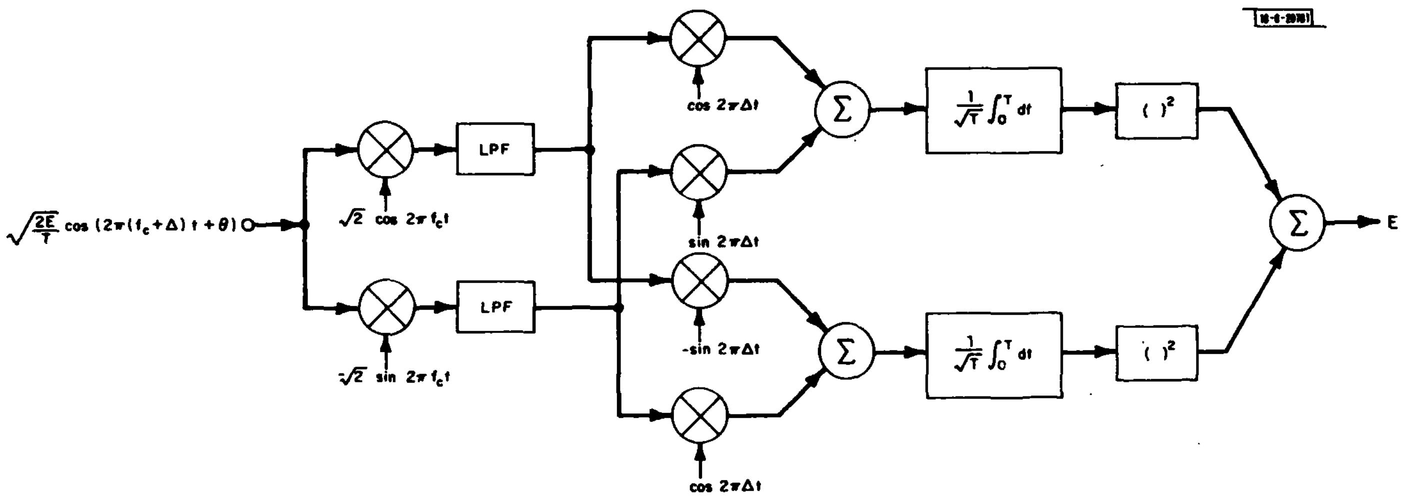

Fig. 1. Receiver structure.

II. PROBABILITY OF ERROR

When $\Delta t > T$ or Δf_c is large, error probabilities may be analyzed as though no interfering signal were present. This analysis for the channel described here may be found in Bernstein [2]. Because some of these results are needed for the subsequent discussion, they are summarized in the next section.

When $\Delta t < T$ and Δf_c is small, the interfering signal will not, in general, be orthogonal to any of the message waveforms and each of the M filters in the receiver will see some splattered energy. This case is too complicated for the probability of error to be computed exactly. However, it may be upper bounded by use of the union bound and the results of section II-B. In that section, the case where $\Delta t = 0$ and $\Delta f_c = 0$ is analyzed. In section II-C, these results are used to bound the probability of error for the general case.

A. No Interfering Signal

Since the messages are all equally likely, it may be assumed without loss of generality that the desired message is m_o. When there is no interfering signal, the filter outputs X_i, $i = 1,\ldots,M-1$, are Gaussian random variables. The distribution for X_0 is given by Lindsey [3] as:

$$p_{X_0}(\alpha) = \begin{cases} \frac{1}{N_o} \left(\frac{\alpha}{E_T}\right)^{\frac{N-1}{2}} e^{-\left(\frac{\alpha+E_T}{N_o}\right)} I_{N-1}\left(\frac{2}{N_o}\sqrt{E_T}\right) & \alpha \geq 0 \\ 0 & \alpha < 0 \end{cases} \tag{2}$$

where E_T is the total energy of the N chip waveforms, and $I_{N-1}(x)$ is the modified Bessel function of order N-1. This is called a noncentral chi-square distribution. When N=1, it is called a Rician distribution.

For binary signaling (M=2), the probability of error is

$$P_2(E) = P_2(E|m_o)$$

$$= P(X_1 > X_0)$$

$$= \int_0^\infty P(X_1 > \alpha) p_{X_0}(\alpha) d\alpha$$

$$= (\tfrac{1}{2})^N e^{-(\frac{E_T}{2N_o})} \sum_{n=0}^{N-1} \binom{N+n-1}{n} (\tfrac{1}{2})^n F(-n, N; -\frac{E_T}{2N_o}) \tag{3}$$

where F(-n, N; X) is the hypergeometric function, given by:

$$F(-n,\ N;\ X) = 1 + \frac{-n}{N(1!)} X + \frac{(-n)(-n+1)}{N(N+1)(2!)} X^2 + \ldots \tag{4}$$

For the LES-8/9 system, N=4 and this becomes:

$$P_2(E) = \frac{1}{16} e^{-(E_T/N_o)} (8 + \frac{29}{8} (\frac{E_T}{2N_o}) + \frac{1}{2} (\frac{E_T}{2N_o})^2 + \frac{1}{48} (\frac{E_T}{2N_o})^3) \tag{5}$$

For M-ary signaling, the union bound gives

$$P(E) \leq (M - 1)\ P_2(E) \quad . \tag{6}$$

B. Interfering Signal with No Time or Doppler Shift

When the interfering signal experiences no time or frequency shift with respect to the desired signal, it will be an amplitude-scaled version of one of the M orthogonal message waveforms. If the interfering message is m_I, then two cases must be considered: $m_I = m_0$ and $m_I \neq m_0$. When $m_I = m_0$, the two waveforms may add constructively or destructively, depending on the difference in phase between them. When $m_I \neq m_0$ and the received energy of the two signals is the same, the receiver will choose between them with equal probability. These effects are discussed quantitatively in the following subsections.

1. Desired and Interfering Signals on the Same Frequency

When $m_I = m_0$, the received signal waveform will be:

$$r(t) = \sqrt{\frac{2E}{NT}} \sum_{i=1}^{N} \cos(\omega_i t + \theta_i) \, rect_T(t - T(i-1)) +$$

$$\sqrt{\frac{2E_I}{NT}} \sum_{i=1}^{N} \cos(\omega_i t + \phi_i) \, rect_T(t - T(i-1)) + n(t) \tag{7}$$

where ω_i is the frequency in rads/sec of the ith chip of the message waveform, θ_i and ϕ_i are the phases of the ith chip waveforms for the desired and unwanted signals, respectively, E and E_I are the received energies of the desired and unwanted signals, and $n(t)$ is a white Gaussian noise process with power density $N_o/2$. The function $rect_T$ (t) is defined by

$$rect_T(t) = \begin{cases} 1 & 0 < t \leq T \\ 0 & \text{elsewhere} \end{cases} . \tag{8}$$

By considering two chip waveforms as phasors (see Fig. 2) and applying some basic trigonometric identities, it is easy to see that:

$$r(t) = \sum_{i=1}^{N} \sqrt{\frac{2E_i'}{T}} \cos(\omega_i \, t + \Psi_i) \, rect_T(t - T(i-1)) + n(t) \tag{9}$$

where $$E_i' = \frac{1}{N} (E + E_I + 2\sqrt{EE_I} \cos(\theta_i - \phi_i)) \tag{10}$$

and Ψ_i satisfies

$$\sqrt{E_i'} \cos \Psi_i = \sqrt{E/N} \cos \theta_i + \sqrt{E_I/N} \cos \phi_i \tag{11}$$

Let $$E' = \sum_{i=1}^{N} E_i' = E + E_I + \frac{2\sqrt{EE_I}}{N} \sum_{i=1}^{N} \cos \alpha_i \tag{12}$$

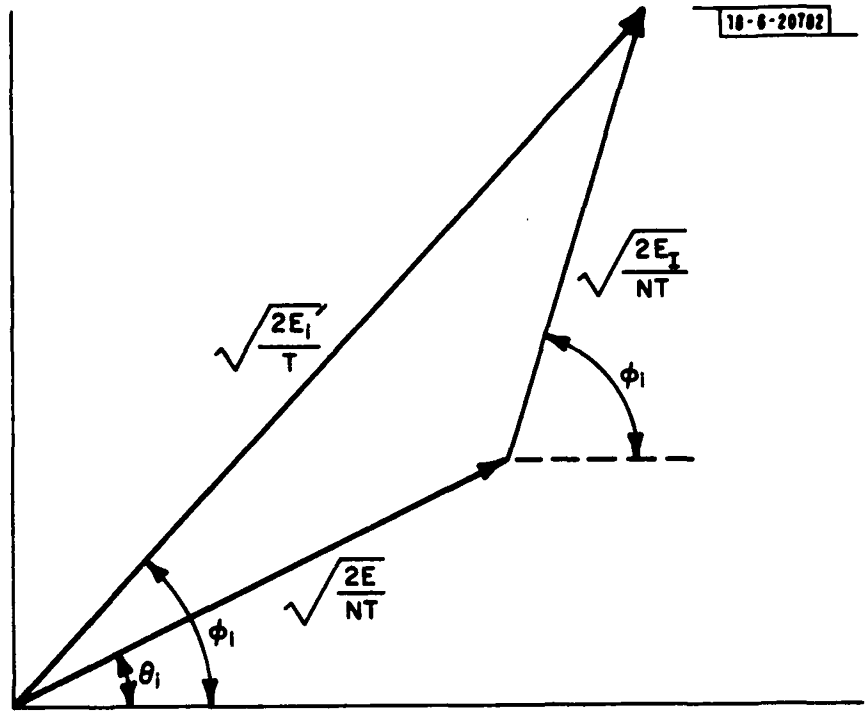

Fig. 2. Phasor diagram for two chips.

where $\alpha_i = \theta_i - \phi_i$. The probability of error for this case will be the same as for no interfering signal and a received signal strength E'. Because E' is a random variable with a complicated distribution, it is not possible to analyze the average probability of error exactly. However, if the random variables α_i are assumed to be uniformly distributed between 0 and 2π, the probability of error is bounded by:

$$P(E) \leq P(E|E' = E + E_I) \quad . \tag{13}$$

A proof of this is given in Appendix A.

It is important to note that this result holds regardless of the distribution of the random variables X_i, $i=1,\ldots,M-1$. Hence it is applicable when there is splattered energy in all the filters, and when there is not.

For an intuitive understanding of this result consider the phasor diagram in Fig. 3. For the i^{th} chip, the amplitude of the desired signal is $\sqrt{\frac{2E}{NT}}$ and its phase is θ_i. A vector of length $\sqrt{\frac{2E_I}{NT}}$ with random phase is added to it. Their sum is a random vector with uniform distribution over the dashed circle shown. Equation (13) asserts that the probability of error will on the average be less than when the phasor of the interfering signal is at right angles to that of the desired signal.

2. Desired and Interfering Signals on Different Frequencies

When $m_I \neq m_0$, the filter outputs X_0 and X_I will have noncentral chi-square distributions, as given by Eq. (2). Let E be the total energy of the desired signal and let E_I be the energy of the interfering signal. Then for binary signaling, the probability of error is

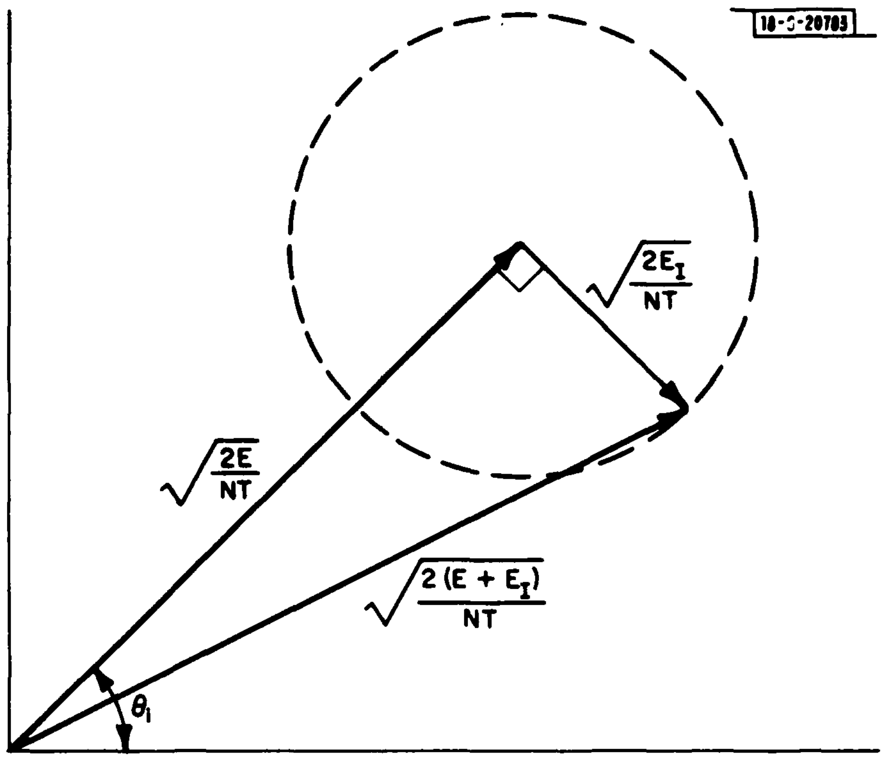

Fig. 3. Phasor diagram of signal plus interference.

$$P_2(E) = P(X_1 > X_0)$$

$$= \int_0^\infty p_{X_0}(\alpha) \int_\alpha^\infty p_{X_1}(\beta)\ d\beta\ d\alpha$$

$$= \int_0^\infty \frac{1}{N_o} \left(\frac{\alpha}{E}\right)^{\frac{N-1}{2}} e^{-(\alpha+E)/N_o}\ I_{N-1}\left(\frac{2}{N_o}\sqrt{E\alpha}\right) .$$

$$Q_N\left(\sqrt{\frac{2E_I}{N_o}}, \sqrt{\frac{2\alpha}{N_o}}\right)\ d\alpha \tag{14}$$

where $Q_N(X,Y)$ is the generalized Q-function defined by Helstrom [4]. That is,

$$Q_N(a,b) = \int_b^\infty x\left(\frac{x}{a}\right)^{N-1} e^{-\left(\frac{x^2+a^2}{2}\right)} I_{N-1}(ax)\ dx \quad . \tag{15}$$

Here, $Q_N\left(\sqrt{\frac{2E_I}{N_o}}, \sqrt{\frac{2\alpha}{N_o}}\right)$ represents the probability that the random variable X_1 exceeds α. In Appendix B it is shown that:

$$P_2(E) = 1 - Q_N\left(\sqrt{\frac{E}{N_o}}, \sqrt{\frac{E_I}{N_o}}\right) +$$

$$\frac{1}{2^N}\ e^{-(E+E_I)/2N_o} \sum_{n=1-N}^{N-1} \left(\frac{E_I}{E_o}\right)^{n/2} I_n\left(\frac{1}{N_o}\sqrt{EE_I}\right) c_n \tag{16}$$

where

$$c_n = \frac{1}{2^{N-1}} \sum_{k=0}^{N-1+n} \binom{2N-1}{k} \tag{17}$$

For the LES-8/9 system, with N=4, Eqs. (16) and (17) give:

$$P(X_1 > X_o) = 1 - Q_4\left(\sqrt{\frac{E}{N_o}}, \sqrt{\frac{E_I}{N_o}}\right) + \frac{1}{2} e^{-\left(\frac{E+E_I}{2N_o}\right)} \cdot$$

$$\left[I_o\left(\frac{1}{N_o}\sqrt{EE_I}\right) + \left(\frac{99}{64}\sqrt{\frac{E_I}{E}} + \frac{29}{64}\sqrt{\frac{E}{E_I}}\right) I_1\left(\frac{1}{N_o}\sqrt{EE_I}\right)\right.$$

$$+ \left(\frac{15}{8}\left(\frac{E_I}{E}\right) + \frac{1}{8}\left(\frac{E}{E_I}\right)\right) I_2\left(\frac{1}{N_o}\sqrt{EE_I}\right)$$

$$\left. + \left(\frac{127}{64}\left(\frac{E_I}{E}\right)^{3/2} + \frac{1}{64}\left(\frac{E}{E_I}\right)^{3/2}\right) I_3\left(\frac{1}{N_o}\sqrt{EE_I}\right)\right] . \qquad (18)$$

This expression can be evaluated by computer, or a programmable calculator with sufficient memory. Methods for computing the generalized Q function may be found in Schnidman [5].

It is gratifying to note that when $E = E_I$, the expression on the right hand side of Eq. (16) equals 1/2, and as E_I goes to 0, it approaches the expression for the case of no interfering signal, as given by Eq. (3).

For M-ary signaling, the union bound may be applied, using Eqs. (3) and (16).

C. Interfering Signal with Time and Doppler Shift

Let m be the desired message and m_I be the message transmitted by the interfering satellite. Suppose that the interfering signal is shifted in time by Δt and in center frequency by Δf_c with respect to the desired signal. Then the i^{th} chip of the interfering signal is shifted in frequency by Δf_{ik} with respect to f_{ik}, the frequency of the i^{th} chip of the k^{th} signal waveform, and Δf_{ik} can be found from the knowledge of m, m_I, Δf_c and f_{ik}. If the i^{th} chip of the interfering signal has energy $E_{I_i} = E_I/N$, then for the receiver structure given in Fig. 1, the interfering energy seen by the i^{th} stage of the k^{th} filter, $k \neq m$, is:

$$E_{I_{ik}} = \frac{2E_{I_i}}{(2\pi\Delta f_{ik}T)^2} \left(1 - \cos[2\pi\Delta f_{ik}(T-\Delta t)]\right), \tag{19}$$

and the total interfering energy in the k^{th} filter is

$$E_{I_k} = \sum_{i=1}^{N} E_{I_{ik}} . \tag{20}$$

When $\Delta f_{ik}=0$, the right side of Eq. (18) is undefined and the limiting form

$$E_{I_{ik}} = \frac{E_{I_i}}{(T-\Delta t)^2} \tag{21}$$

is used.

The energy in the m^{th} filter is a function of both the energy of the desired signal and of the interfering signal, as discussed in Section II.B.1. In that section it is shown that the probability of error for binary signalling is upper-bounded by the probability of error given by assuming that the energy in the m^{th} filter is just the sum of the desired and interfering energies. Hence, let

$$E_m = E + E_{I_m} , \tag{22}$$

where E is the energy of the desired signal and E_{I_m} is given by Eqs. (18) and (19). Then the union bound gives

$$P(E|m, m_I) \leq \sum_{\substack{k=0 \\ k\neq m}}^{M-1} P_2(E|E_m, E_{I_k}) \tag{23}$$

where $P_2(E|E_m, E_{Ik})$ evaluated as in Eq. (16). Since the desired and interfering messages are each assumed to be chosen with equal probability from a set of M messages, the overall probability of error is

$$P(E) = \frac{1}{M^2} \sum_{M=0}^{M-1} \sum_{M_I=0}^{M-1} P(E|m, m_I)$$

$$\leq \frac{1}{M^2} \sum_{M=0}^{M-1} \sum_{M_I=0}^{M-1} \sum_{\substack{k=0 \\ k\neq m}}^{M-1} P_2(E|E_m, E_{I_k}). \tag{24}$$

Equation (23) can be analyzed by computer for various values of Δf_c and Δt. Results will, of course, vary with different values of N, M, E, E_I and the particular code used, that is, the sequence of chip waveforms used to represent a given message. As will be seen in the following section, the choice of code can be very important.

D. The Effect of Coding on Error Probabilities

The code, or the set of sequences of waveforms used to represent the different messages, can make a significant difference in the error performance in the presence of an interfering signal. To understand why this is so, consider the following two codes as they might be applied to the LES-8/9 system.

As described in the introduction, the LES-8/9 system uses as its chip waveforms a set of 8 sinusoids. Denote each of these by a number from 0 to 7. The numbering may be arbitrary, but it is convenient to let f_c + 700 Hz be represented by 0, f_c + 500 Hz by 1, etc. Consider the code where the message m=0 is sent by transmitting four repetitions of the waveform 0, m = 1 is sent by four repetitions of the waveform 1, and so on. This code may be represented by the following matrix:

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 \\ 4 & 4 & 4 & 4 \\ 5 & 5 & 5 & 5 \\ 6 & 6 & 6 & 6 \\ 7 & 7 & 7 & 7 \end{bmatrix} .$$

This code has the obvious disadvantage that when Δf_c is a small integer multiple of 200 Hz, in most cases all of the energy in the interfering signal will fall into a single filter, resulting in a high probability of error. Even when Δf_c is not a multiple of 200 Hz, most of the interfering energy will be seen by two adjacent filters. It should be clear that for the best error performance, it is desirable to spread the interfering energy between the filters as uniformly as possible.

The code used by LES-8/9 has this property. It is known as the Queen's code [6] and is described by the matrix

$$\begin{bmatrix} 3 & 0 & 7 & 4 \\ 1 & 1 & 6 & 6 \\ 6 & 2 & 5 & 1 \\ 2 & 3 & 4 & 5 \\ 5 & 4 & 3 & 2 \\ 7 & 5 & 2 & 0 \\ 0 & 6 & 1 & 7 \\ 4 & 7 & 0 & 3 \end{bmatrix}$$

Suppose, for example, that the interfering message is m = 1, represented by the codeword (1,1,6,6), but is received shifted by +200 Hz, represented by (2,2,7,7). This distributes the interfering energy equally between the filters for the messages 3,2,0 and 6. It is easily verified that for any of the eight possible interfering messages and for any frequency shift which is a multiple of 200 Hz, this property holds.

A computer program was written to analyze the probability of error expression in Eq. 23 for these two codes, for a variety of signal energies and time and frequency shifts. The signal to noise ratio for both signals was assumed to be the same. This is a reasonable assumption since the two satellites must be approximately equi-distant from the receiver in order that Δt be less than 5 msec and because the receiver antenna is non-directional. The probability of character error was converted to bit error and an operating threshold of $p_b = .028$ was used. It was found that when E_b/N_o is less than 8 dB, the system will not function for either code, for any time or frequency shift. For $E_b/N_o = 8$ dB using the first code, the system will not function for Δt less than 1 msec for any Δf_c, will not function for Δt between 1 msec and 2 msec for Δf_c less than 600 Hz, but functions acceptably for any Δf_c when Δt is 2 msec or more. Using the Queen's code, however, the system fails only when Δt is less than 1 msec and Δf_c less than 150 Hz, or Δt is between 1 msec and 2 msec and Δf_c is less than 100 Hz! Results for higher signal to noise ratios are similar, but not as dramatic.

In addition to having low correlation between shifted and unshifted code words, there is another property which, if incorporated in a code, should improve performance. When Δf_c is not a multiple of 200 Hz, most of the interfering energy will be seen by the filters whose frequencies are closest to the frequencies of the interfering signal. If Δf_c is small, and the total distances between the interfering codword and some other codeword is small, the filter for that codeword will see much of the interfering energy. For example, if the interfering codeword is (3,0,7,4), the filter for the codeword (1,1,6,6) will see more of the interfering energy than, say, the filter for (7,5,2,0). Hence, it is desired to have the distance between any pair of codewords as great as possible. A computer search was made for such a code, but preliminary investigations show that the improvement in performance over the Queen's code is negligible.

E. Probability of Error for Three Satellites

If the receiver is within range of three satellites it will, in general, see some energy from the two interfering signals in all of its filters, in addition to the energy of the desired signal in the appropriate filter. The interfering signals in all the filters may add constructively or destructively as described in section II.B.1. It would be convenient to simply sum the energies of the interfering signals for each filter and use these values in Eq. 23 to find an upper bound to the probability of error. This would not, however, be correct. In Appendix A it is shown that if the random variable X_0 is the output of a filter receiving two interfering signals, then

$$P(X_1 < X_0) \geq P(X_1 < X_0 | E' = E_1 + E_2), \tag{25}$$

where X_1 is any random variable, E_1 and E_2 are the energies of the two signals, and E' is the energy of the sum of the two signals. If the event $X_0 > X_1$ implies a correct decision, as when X_0 is the output of the desired message's filter, then clearly this implies

$$P_2(\mathcal{E}) \leq P_2(\mathcal{E} | E' = E_1 + E_2) \quad . \tag{26}$$

However, if the event $X_0 > X_1$ implies an incorrect decision, as when X_0 is the output of any other filter, Eq. 24 gives

$$P_2(E) \geq P_2(E|E' = E_1 + E_2) \quad . \tag{27}$$

One way to upper bound the probability of error is to use the worst case assumption that the unwanted signals always add in phase, resulting in the strongest possible interference. That is,

$$P_2(E) \leq P_2(E|E' = E_1 + E_2 + 2\sqrt{E_1 E_2}) \quad . \tag{28}$$

This expression, however, is overly conservative and when combined with the union bound in Eq. 23, yields an upper bound which is too loose to be of value.

III. CONCLUSIONS

For a system of two satellites using a suitably chosen code, as described in Section II.D., it is strongly indicated that the same frequency hopping pattern may be used by both without significant performance degradation. Of course, any specific system must be analyzed with respect to its own performance requirements (minimum acceptable probability of bit error) and geometry, to insure that the receiver rarely falls into the critical regions of Δt and Δf_c. It is important to keep in mind that these results are, in fact, conservative, due to the use of the union bound, and that the system will probably function even better than predicted.

APPENDIX A

To prove Eq. (13), consider the probability that the receiver decodes the signal correctly. Then

$$P(C|E') = \int_0^\infty \frac{1}{N_o} \left(\frac{x}{E'}\right)^{N-1} e^{-\left(\frac{x+E'}{N_o}\right)} I_{N-1}\left(\frac{2}{N_o}\right)\sqrt{xE'})$$

$$\cdot \; P(X_i \leq x,\ i = 1,\ldots,M-1)\ dx \qquad \text{(A.1)}$$

and

$$P(C) = \int_0^\infty P(C|z) p_{E'}(z)\ dz$$

$$= \frac{1}{(2\pi)} N \int_0^{2\pi} \cdots \int_0^{2\pi} \int_0^\infty \frac{1}{N_o} \left(\frac{x}{E+E_I + \frac{2\sqrt{EE_I}}{N} \sum_{i=1}^N \cos\alpha_i} \right) \cdot$$

$$\exp\left(-(x + E + E_I + \frac{2\sqrt{EE_I}}{N} \sum_{i=1}^N \cos\alpha_i)/N_o\right) \cdot$$

$$I_{N-1}\left(\frac{2}{N_o}(x(E + E_I + \frac{2\sqrt{EE_I}}{N} \sum_{i=1}^N \cos\alpha_i))^{1/2}\right) \cdot$$

$$P(X_i \leq x,\ i = 1,\ldots,M - 1)\ dx\ d\alpha_1 \ldots d\alpha_N \qquad \text{(A.2)}$$

Now define:

$$f(E) = E^{1-N} e^{-E/N_o} I_{N-1}\left(\frac{2}{N_o}\sqrt{xE}\right) \qquad \text{(A.3)}$$

for any $x \geq 0$, $E \geq 0$. Then algebraic manipulations can be used to show that $f(E)$ is a convex function. Therefore

$$f(E + k) + f(E - k) \geq 2f(E) \qquad \text{(A.4)}$$

for any k such that E-k is positive. Returning to the integral, it is seen that

$$P(C) = \frac{1}{N_o \pi^N} \int_0^\infty X^{N-1} e^{-(x/N_o)} P(X_i \leq x,\ i = 1,\ldots,M - 1) \cdot$$

$$\int_0^\pi \cdots \int_0^\pi \int_0^{\pi/2} [(E + E_I + \frac{2\sqrt{EE_I}}{N} \sum_{i=1}^{N} \cos\alpha_i)^{1-N}] \cdot$$

$$\exp(-(E + E_I + \frac{2\sqrt{EE_I}}{N} \sum_{i=1}^{N} \cos\alpha_i)/N_o) I_{N-1}(\frac{2}{N_o}(x(E + E_I + \frac{2\sqrt{EE_I}}{N_o} \sum_{i=1}^{N} \cos\alpha_i))^{1/2})$$

$$+ (E + E_I + \frac{2\sqrt{EE_I}}{N} \sum_{i=2}^{N} \cos\alpha_i - \frac{2\sqrt{EE_I}}{N} \cos\alpha_1)^{1-N} \cdot$$

$$\exp(-(E + E_I + \frac{2\sqrt{EE_I}}{N} \sum_{i=2}^{N} \cos\alpha_i - \frac{2\sqrt{EE_I}}{N} \cos\alpha_1)/N_o) \cdot$$

$$I_{N-1}(\frac{2}{N_o} (x(E + E_I + \frac{2\sqrt{EE_I}}{N} \sum_{i=2}^{N} \cos\alpha_i - \frac{2\sqrt{EE_I}}{N} \cos\alpha_1))^{1/2})$$

$$\cdot\ d\alpha_1 \ldots d\alpha_N dx \tag{A.5}$$

Combining (A.4) and (A.5) gives:

$$P(C) \geq \frac{1}{N_o \pi^{N-1}} \int_0^\infty x^{N-1} e^{-(x/N_o)} P(X_i \leq x,\ i = 1,\ldots,M - 1) \cdot$$

$$\int_0^\pi \cdots \int_0^\pi (E + E_I + \frac{2\sqrt{EE_I}}{N} \sum_{i=2}^{N} \cos\alpha_i)^{1-N} \cdot$$

$$\exp(-(E + E_I + \frac{2\sqrt{EE_I}}{N} \sum_{i=2}^{N} \cos\alpha_i)/N_o) \cdot$$

$$I_{N-1}(\frac{2}{N_o} (x(E + E_I + \frac{2\sqrt{EE_I}}{N} \sum_{i=2}^{N} \cos\alpha_i))^{1/2})$$

$$\cdot\ d\alpha_2 \ldots d\alpha_N dx \tag{A.6}$$

The random variable α_1 has been eliminated from the inequality. Repeating this process, α_2 through α_N may also be eliminated to give:

$$P(C) \geq \frac{1}{N_o} \int_0^\infty \left(\frac{x}{E+E_I}\right)^{N-1} e^{-(x+E+E_I)/N_o} I_{N-1}\left(\frac{2}{N_o}\sqrt{x(E+E_I)}\right)$$

$$\cdot\ P(X_i \leq x,\ i = 1,\ldots,M-1)\ dx$$

$$= P(C|E' = E + E_I) \tag{A.7}$$

Finally,

$$P(\mathcal{E}) \leq P(\mathcal{E}|E' = E + E_I) \tag{A.8}$$

Q.E.D.

APPENDIX B

To evaluate the integral

$$I = \int_0^\infty Q_N(b,x)x\left(\frac{x}{a}\right)^{N-1} e^{-\left(\frac{x^2+a^2}{2}\right)} I_{N-1}(ax)dx \qquad (B.1)$$

first note

$$Q_N(a,b) = e^{-\left(\frac{a^2+b^2}{2}\right)} \sum_{n=1-N}^{\infty} \left(\frac{a}{b}\right)^n I_n(ab) \quad . \qquad (B.2)$$

This can be shown by replacing the Bessel function in the definition of the Q function by its infinite series, and integrating. Hence,

$$Q_N(a,b) = 1 + e^{-\frac{a^2+b^2}{2}} \sum_{n=1-N}^{N-1} \left(\frac{b}{a}\right)^n I_n(ab) - Q_N(b,a) \qquad (B.3)$$

Substituting (B.3) into (B.1) gives

$$I = \int_0^\infty [1 + e^{-\left(\frac{x^2+b^2}{2}\right)} \sum_{n=1-N}^{N-1} \left(\frac{x}{b}\right)^n I_n(bx) - Q_N(x,b)] \cdot$$

$$x\left(\frac{x}{a}\right)^{N-1} e^{-\left(\frac{x^2+a^2}{2}\right)} I_{N-1}(ax)dx. \qquad (B.4)$$

This can be broken into three integrals which may be evaluated separately. First,

$$I_1 = \int_0^\infty x\left(\frac{x}{a}\right)^{N-1} e^{-\left(\frac{x^2+a^2}{2}\right)} I_{N-1}(ax)dx = Q_N(a,0) = 1 \qquad (B.5)$$

Next,

$$I_2 = \int_0^\infty Q_N(x,b)x(\frac{x}{a})^{N-1} e^{-\left(\frac{x^2+a^2}{2}\right)} I_{N-1}(ax)dx$$

$$= Q_N(\frac{a}{\sqrt{2}}, \frac{b}{\sqrt{2}}) \; . \qquad \text{(B.6)}$$

This follows from Eq. (15) of Nuttall [7]. Finally, it is necessary to evaluate

$$I_3 = e^{-(\frac{a^2+b^2}{2})} \sum_{n=1-N}^{N-1} \frac{1}{b^n a^{N-1}} \int_0^\infty e^{-x^2} x^{N+n} I_{N-1}(ax)I_n(bx)dx \qquad \text{(B.7)}$$

To do this, consider the function f(N,M,K) defined by:

$$f(N,M,K) = \int_0^\infty e^{-x^2} x^K I_N(ax)I_M(bx)dx \; . \qquad \text{(B.8)}$$

If K=N+M+1, f(N,M,K) may be evaluated as follows. Using the method of integration by parts, with

$$\mu = x^M e^{-x^2} I_M(bx)$$

and

$$dv = x^{N+1} I_N(ax)dx,$$

it is easy to show that:

$$f(N,M,N+M+1) = \frac{2}{a}\int_0^\infty x^{N+M+2}\, e^{-x^2}\, I_{N+1}(ax)I_M(bx)dx$$

$$- \frac{b}{a}\int_0^\infty x^{N+M+1}\, I_{N+1}(ax)I_{M-1}(bx)dx$$

$$= \frac{2}{a}\, f(N+1,M,N+M+2) - \frac{b}{a}\, f(N+1,M-1,N+M+1) \qquad (B.9)$$

Hence,

$$f(N,M,N+M+1) = \frac{a}{2}\, f(N-1,M,N+M) + \frac{b}{2}\, f(N,M-1,N+M) \qquad (B.10)$$

Now, by induction on N+M, it can be shown that

$$f(N,M,N+M+1) = \frac{1}{2^{N+M}} \sum_{n=-M}^{M} \binom{N+M}{N-n} a^{N-n}\, b^{M+n}\, f(n,n,1) \qquad (B.11)$$

for any N,M such that $N+M \geq 0$. To see this, first note that, by the definitions of f(N,M,K) and the modified Bessel functions,

$$f(N,-N,1) = f(N,N,1)\ . \qquad (B.12)$$

Hence (B.11) holds for any N,M such that N+M = 0. Now suppose that (B.11) holds for any N,M such that N+M=K. Then for any N,M such that N+M=K+1, (B.11) must hold for f(N,M-1,N+M) and f(N-1,M,N+M). Hence, by (B.10)

$$f(N,M,N+M+1) = \frac{a}{2}\, \frac{1}{2^{N+M-1}} \sum_{n=-M}^{N-1} \binom{N-1+M}{N-n-1} a^{N-1-n}\, b^{M+n}\, f(n,n,1)$$

$$+ \frac{b}{2}\, \frac{1}{2^{N+M-1}} \sum_{n=-M+1}^{N} \binom{N-1+M}{N-n} a^{N-n}\, b^{M-1+n}\, f(n,n,1)$$

$$= \frac{1}{2^{N+M}} \sum_{n=-M}^{N} \binom{N+M}{N-n} a^{N-n}\, b^{M+n}\, f(n,n,1),$$

which establishes (B.11).

Finally, from integral (5), Section 6.633, Gradshteyn and Rhyzik [8].

$$f(N,N,1) = \frac{1}{2} e^{\left(\frac{a^2+b^2}{4}\right)} I_N\left(\frac{ab}{2}\right) \quad . \tag{B.13}$$

Combining Eqs. (B.7), (B.8), (B.11) and (B.13) gives

$$I_3 = e^{-\left(\frac{a^2+b^2}{2}\right)} \sum_{n=1-N}^{N-1} \frac{1}{b^n a^{N-1}} f(N-1,n,N+n)$$

$$= \frac{1}{2^N} e^{-\left(\frac{a^2+b^2}{4}\right)} \sum_{n=1-N}^{N-1} \frac{1}{2^n} \sum_{k=-n}^{N-1} \binom{N+n-1}{n+k} \left(\frac{b}{a}\right)^k I_k\left(\frac{ab}{2}\right)$$

$$= \frac{1}{2^N} e^{-\left(\frac{a^2+b^2}{4}\right)} \sum_{n=1-N}^{N-1} \left(\frac{b}{a}\right)^n I_n\left(\frac{ab}{2}\right) \sum_{k=-n}^{N-1} \binom{N-1+k}{n+k} 2^{-k} \tag{B.14}$$

By induction, it can be shown that

$$\sum_{k=-n}^{N-1} \binom{N-1+k}{n+k} = 2^{N-1} \sum_{k=0}^{N-1+n} \binom{2N-1}{k} \tag{B.15}$$

Combining Eqs. (B.4), (B.5), (B.6), (B.14) and (B.15) gives the desired result:

$$I = 1 - Q_N\left(\frac{a}{\sqrt{2}}, \frac{b}{\sqrt{2}}\right) + \frac{1}{2^N} e^{-\left(\frac{a^2+b^2}{4}\right)} \sum_{n=1-N}^{N-1} \left(\frac{b}{a}\right)^n I_n\left(\frac{ab}{2}\right) c_n \tag{B.16}$$

where

$$c_n = 2^{N-1} \sum_{k=0}^{N-1+n} \binom{2N-1}{k} \quad . \tag{B.17}$$

Q.E.D.

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This report considers a situation in which more than one satellite transmitting frequency-hopped signal is in view of a given terminal. If it is desired that the satellites use the same frequency hopping pattern for ease of acquisition and handover, then a receiver which is in range of more than one satellite may encounter interference. This interference is analyzed for a generalized system of two satellites modelled after the LES-8/9 system. Upperbounds for the probability of error are derived for receivers using a square-law combining rule. This expression is evaluated for the LES-8/9 system, using two different downlink codes. It is seen that the choice of code makes a significant difference in system performance, and that with a suitably chosen code, two satellites may use the same frequency hopping pattern without significant performance degradation.

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